

Course No	Course Name	L-T-P-Credits
MA 408	Measure Theory and Integration	3-1-0: 4

Prerequisite: nil

**Course Objectives:** Measure Theory provides a foundation for many branches of mathematics such as probability theory, stochastic process and functional analysis. The aim of this course is to learn the basic elements of Measure Theory providing the students an additional opportunity to develop skills in modern analysis as well as providing a rigorous foundation for other branches of mathematics.

**Course Outcomes:** After successful completion of the course, students will be able to:

1. Understand sigma algebra and measurable sets and measurable functions.
2. Understand the fundamentals of measure theory and be acquainted with the proofs of the fundamental theorems underlying the theory of integration.
3. Understand the need of the measure and integration.
4. Demonstrate the understanding of Lebesgue integral and be able to distinguish it from Riemann integral.
5. Gain the knowledge of basic properties of  $L_p$  spaces.
6. Develop a perspective on the broader impact of measure theory in different fields.

### SYLLABUS

Module	Contents	Hours
I	Measure: Sigma-algebra, Measures, measurable sets, Lebesgue outer measure, Lebesgue measure, Borel set, Borel sigma-algebras, Borel measure.	09
II	Measurable functions: Measurability, pointwise convergence, almost everywhere convergence, Egoroff's theorem, Lusin's theorem.	09
III	Integration: Integration of simple functions, positive functions, measurable functions; Fatou's lemma, convergence theorems; signed measures, Hahn and Jordan decomposition, Radon-Nikodym theorem.	12
IV	$L_p$ spaces: $L_p$ spaces, Minkowski and Hölder inequalities, Density.	06

**Essential Readings:**

1. H. L. Royden and P. M. Fitzpatrick, “*Real Analysis*”, Pearson Education India, 4<sup>th</sup> edition, 2015.
2. D. L. Cohn, “*Measure Theory*”, Birkhäuser, 2<sup>nd</sup> edition, 2013

**Supplementary Readings:**

1. E. M. Stein and R. Shakarchi, “*Real analysis: Measure Theory, Integration, and Hilbert Spaces*”, New Age International Private Limited, 1st edition, 2010.
2. G. De Barra, “*Measure Theory and Integration*”, New Age International Private Limited, 1st edition, 2013.