

Course No	Course Name	L-T-P-Credits
<b>MA 535</b>	<b>Advanced Number Theory</b>	<b>3-0-0: 3</b>

Prerequisite: NIL

**Course Objectives:** Number theory is strongly connected with many other branches of mathematics. The objective of the course is to present a balance view of the subject to gain insights into the many branches of number theory.

**Course Outcomes:** After successful completion of the course, students will be able to:

1. Understand the concept of congruences and use various results related to congruences including the Chinese Remainder Theorem and congruences of higher degree.
2. Understand the concept of quadratic Gaussian sums and reciprocity.
3. Know about Gauss and Jacobi Sums and their applications.
4. Solve certain types of Diophantine equations.
5. Know writing real number in continued fraction and applications involving continued fraction.

### SYLLABUS

Module	Contents	Hours
I	Review of congruences, Euler's function, results of Fermat, Euler and Wilson; linear congruences, Chinese remainder theorem. Primitive roots and the group structure of $U(\mathbb{Z}/n\mathbb{Z})$ ; applications to congruences of higher degree.	8
II	Quadratic Gaussian Sums and Reciprocity: Quadratic Residues, Gaussian reciprocity law, the Jacobi symbol, Quadratic Gauss Sums, Sign of the Quadratic Gauss Sum.	8
III	Gauss and Jacobi Sums: Finite Field and its properties, Gauss Sums, Jacobi sum, the equation $x^n + y^n = 1$ .	8
IV	Diophantine equations. Linear equations, the equation $x^2 + y^2 = z^2$ . Method of Descent; the equation $x^4 + y^4 = z^2$ .	6
V	Simple continued fractions. Infinite continued fractions and irrational numbers. Periodic continued fractions.	6

#### Essential Readings:

1. S K. Ireland and M. Rosen, "A Classical Introduction to Modern Number Theory", Springer, 2<sup>nd</sup> edition, 2009
2. I. Niven and H. S. Zuckerman, "An Introduction to the Theory of Numbers", Wiley, 5th edition, 2005

#### Supplementary Readings:

1. J. H. Silverman, "A Friendly Introduction to Number Theory", Pearson Education India, 4<sup>th</sup> edition, 2014.
2. D. M. Burton, "Elementary Number Theory", McGraw Hill Education, 7<sup>th</sup> edition, 2017