Course No	Course Name	L-T-P-Credits
MA 546	**Rings and Modules**	**3-1-0:4**

Prerequisite: Abstract Algebra

Course Objectives: To motivate the students for research in ring theory by exploring some advanced concept of rings and modules.

Course Outcomes: Upon successful completion of the course, students will be able to:

1. Be fluent in fundamental concept of rings.
2. Be fluent in concept of ideals and their operations.
3. Compute nil radical, Jacobson radical and singular ideal of standard rings.
4. Know basic properties of polynomial rings, matrix rings and regular rings
5. Be fluent in fundamental concept of modules.
6. Be fluent in concept of submodules and their operations.
7. Set up exact and short exact sequences of modules and know some basic properties of the sequences.
8. Be fluent with the concept of direct sum, direct product and tensor product of modules.
9. Know fundamental properties of classes of modules like injective modules, projective modules, divisible modules, simple modules, semisimple modules, flat modules, free modules and should be able to give standard examples of each class.
10. Apply the knowledge and skills obtained to create and investigate some research problem in ring theory.

SYLLABUS

Module	Contents	Hours
I	Basic concepts of rings, ideals, matrix rings, polynomial rings, direct products of rings, fields and division rings, domains, regular rings, idempotent and nilpotent elements in a ring. Modules, submodules, operation on submodules, large and small submodules, radical of a module.	10
II	Homomorphism of modules, isomorphism theorems, exact sequences, direct sums and direct products of modules, external and internal direct sums, direct summands.	6
III	Free modules and projective modules, torsion free and torsion modules over commutative domains, exact sequences and projectivity.	6
IV	Tensor product of modules, flat modules, injective modules, injectivity and divisibility over domains, exact sequences and injectivity, Baer's theorem.	7
V	Simple modules, semisimple modules, socle of a module, semisimple rings, Schur's lemma,equivalent conditions for semisimple modules, Wedderburn structure theorem.	7

Essential Readings:

1. N. Jacobson, "Basic Algebra II", Dover Publications Inc, 2nd edition, 2009.
2. S. Lang, "Algebra", Springer, 3rd edition, 2010

Supplementary Readings:

1. I. S. Luthar and I. B. S. Passi, “Algebra, Vol. 2: Rings”, Narosa Publishing House, 2000.
2. J. J. Rotman, “An Introduction to Homological Algebra”, Springer.
3. I. T. Adamson, “Elementary Rings and Modules”, Oliver and Boyd Publisher.